Noble-Gas Binary Mixtures for Closed-Brayton-Cycle Space Reactor Power Systems

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The advantages and limitations of pure helium and noble-gas binary mixtures as working fluids for space nuclear reactor power systems with closed Brayton cycles are examined. Helium has the best transport properties, but its small molecular weight increases the aerodynamic loading of the impeller blades. The binary mixture of He-Xe with a molecular weight of 40 g/mol is the recommended working fluid because it has the same heat transfer coefficient as helium, but results in a significant reduction in the aerodynamic loading (10% of that with helium) and hence, in the size and mass of the turbomachines and the power system. The performance parameters of a power system with no single-point failure in reactor cooling, three independent closed-Brayton-cycle loops, and He-Xe binary-mixture working fluid (40 g/mol) are presented and compared with those reported for similar systems.

Nomenclature

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\boldsymbol{A}	=	cross-sectional flow area, m ²
A_o	=	heat transfer area, m ²
C_P	=	specific heat capacity at constant pressure, J/kg · K
\hat{C}_P	=	molar heat capacity at constant pressure, J/mol · K
C_v	=	specific heat capacity at constant volume, J/kg · K
$\stackrel{\circ}{C_P}$ $\stackrel{\circ}{C_P}$ $\stackrel{\circ}{C_v}$ $\stackrel{\circ}{D}$	=	diameter or equivalent hydraulic diameter, m
f	=	Darcy-Weisbach friction factor
H	=	enthalpy, J/kg
h	=	heat transfer coefficient, $W/m^2 \cdot K$
k	=	thermal conductivity, W/m · K
L	=	length of the flow channel, m
M	=	molecular weight, kg/mol
\dot{m}	=	mass flow rate, kg/s
\dot{N}	=	molar flow rate, mol/s
Nu	=	Nusselt number, hD/k
P	=	pressure, Pa
Pr	=	Prandtl number, $\mu C_P/k$
Q	=	thermal power, W
R	=	impeller tip radius, m
R	=	gas constant, J/kg · K
Re	=	Reynolds number, $(MND)/(\mu A)$
R_g	=	universal gas constant, 8.31441 J/ mol·K
r	=	compressor pressure ratio, $P_{2,o}/P_{1,o}$
T	=	temperature, K
U	=	impeller tip velocity, m/s
U_o	=	overall heat transfer coefficient, $W/m^2 \cdot K$
\dot{W}	=	rate of mechanical energy, W
Z	=	gas compressibility factor
Z	=	distance from the channel entrance, m
α	=	recuperator exponential coefficient
β	=	fraction mass flow rate in the bleed line
γ	=	specific heat ratio, C_p/C_v
ΔH_C	=	enthalpy increase in the compressor, J/kg
ΔH_T	=	enthalpy decrease in the turbine, J/kg
ΔP	=	total pressure losses, Pa
ε	=	recuperator effectiveness
η	=	net system efficiency

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 $\eta_C = \text{compressor polytropic efficiency}$ $\eta_G = \text{generator electrical efficiency}$

 η_G = generator electrical efficiency η_M = closed-Brayton-cycle shaft mechanical efficiency

 η_T = turbine polytropic efficiency

 θ = cycle stagnation-temperature ratio, $T_{4,o}/T_{1,o}$

 λ_C = compressor aerodynamic loading μ = dynamic viscosity, kg/m·s π = loop pressure-loss factor ρ = density, kg/m³

 ω = shaft angular speed, rad/s

Subscripts

b bulk Ccompressor Ggenerator He helium Kr krypton stagnation or total 0 R recuperator rad heat-rejection radiator RX nuclear reactor

T = turbine w = wall Xe = xenon 1 = compressor inlet 2 = compressor exit

3 = nuclear reactor inlet
4 = turbine inlet
5 = turbine exit
6 = gas cooler inlet
7 = nuclear reactor exit
8 = coolant bleed-line exit

Superscripts

He = helium

I. Introduction

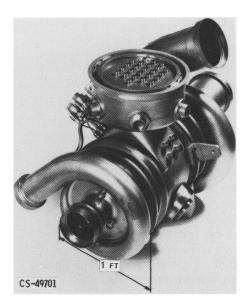
S PACE nuclear reactor power systems with either static or dynamic energy conversion are viable choices for faster travel to the farthest planets, for which the solar option is nonexistent, as well as for ample power for planetary exploration and lunar and Mars outposts. These power systems, designed to operate safely and reliably for many years, do not start up until reaching their destination or after deployment into a flight trajectory. They operate continuously, or intermittently as needed by the mission profile, independently of the sun, for 10–15 years and even longer and can

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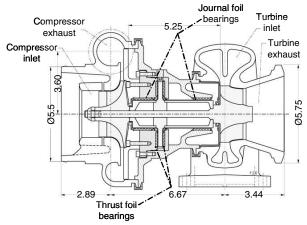
generate hundreds to thousands of kilowatts of electrical power for housekeeping and/or powering multiple ion propulsion engines for faster travel to distant planets. The choices of the energy-conversion technology not only affect the design, operation and integration of space nuclear reactor power systems, but also their stowed volume and mass, launch cost, type of launch vehicle, and operational characteristics.

A closed Brayton cycle (CBC) with single-shaft, centrifugal-flow turbomachines [1–4] is an attractive energy-conversion option with a gas-cooled reactor heat source. These turbomachines (Fig. 1) are compact and of low mass for generation of hundreds of kilowatts to tens of megawatts of electrical power for either space or terrestrial applications. By contract, terrestrial nuclear reactor power plants, for power levels greater than 100-150 MWe, typically employ multipleshaft turbomachines [5,6]. The major difference between space and terrestrial uses of CBC engines for electrical power generation is the emphasis in the former on compactness and low specific mass (kg/ kWe), which favors using compact and low-mass turbomachines and operating close to the peak electric power of the cycle. For terrestrial applications, however, in which the emphasis is on high electric power levels and low production cost, the nuclear power plants use multiple-shaft, axial-flow turbomachines and operate nominally at or close to the peak conversion efficiency.

Because of their chemical inertness, noble gases are the working fluids of choice for space power systems and terrestrial power plants employing gas-cooled nuclear reactors and CBC engines. Helium



a) Photograph of a BRU



b) Typical dimensions of a BRU

Fig. 1 $\,$ A 10.5-kWe BRU [1] with oil-free, gas foil bearings (dimensions are in inches).

(He), with the lowest molecular weight, has the best thermal and transport properties of all pure noble gases and is the preferred working fluid for terrestrial power plants [5,6]. Conversely, the small molecular weight of He increases the aerodynamic loading of the impeller blades and hence, the mass and the size of the turbomachines for space applications, or the number of stages in the axial-flow compressor and turbine units in terrestrial power plants. The higher-molecular-weight noble gases of neon (Ne), argon (Ar), krypton (Kr), and xenon (Xe) have significantly less favorable thermal and transport properties compared with He, but their use as working fluids decreases the aerodynamic loading and the size and/or the number of stages of the turbomachines. When mixing He with heavier noble gases such as Xe and Kr, the properties of the binary mixtures are superior to those of pure gases of an equal molecular weight [7].

The objective of this paper is to investigate the attributes and limitations of pure noble gases and binary mixtures as working fluids in space nuclear reactor power systems with CBC loops. These systems typically operate at \sim 2.0 MPa and reactor and compressor exit temperatures of less than or equal to 1200 and 450 K, respectively. Compared are the thermal and transport properties of the noble gases and their binary mixtures, along with the relative values of the turbulent heat transfer coefficients, aerodynamic loading of the compressor impeller and turbine blades in the turbomachines, and the pressure losses. The thermal properties of the noble gases and their binary mixtures are determined using an extended kinetic theory approach developed for dense gases and mixtures, based on the two-parameter law of corresponding states [7]. This approach was benchmarked successfully using an extensive properties database for temperatures up to 1400 K and pressures up to 20 MPa. It showed that unlike heavy noble gases of Xe and Kr, pure helium always behaves as a dilute gas, for which the transport properties are independent of pressure [7].

The calculated thermal conductivity, dynamic viscosity, gas compressibility, molar specific heat, specific heat ratio, and Prandtl number of all noble gases and their binary mixtures at 2.0 MPa and both 400 and 1200 K are discussed and compared. These temperatures represent the lowest and highest values of the working fluid in the space nuclear reactor power system developed and presented in this paper. This power system [8] has no single-point failures in reactor cooling or the CBC energy conversion (Fig. 2), three independent CBC loops, a sectored gas-cooled nuclear reactor, and He-Xe working fluid with a molecular weight of 40 g/mol. The performance results are compared with those of similar space nuclear power systems by other investigators. The Appendix at the end of the paper presents a brief description of the model used to calculate the performance parameters, including the net system efficiency and load electrical power.

II. Analysis

This section develops analytical expressions for the relative heat transfer coefficient and pressure losses for a turbulent flow of pure noble gases and their binary mixtures, as well as for the aerodynamic loading of the compressor impeller blades. Normalized values are compared for the same molar flow rate and geometrical dimensions and/or pressure ratio as functions of the molecular weight of the working fluid at typical operating pressure (2.0 MPa) and temperatures in space nuclear reactor power systems with CBC engines.

A. Heat Transfer Coefficient

The turbulent heat transfer coefficient calculated using the correlation proposed by Taylor [9] was shown to agree with experimental data for gases at low heating rates to within $\pm 10\%$ and to within $\pm 20\%$ at high heating rates [10]. According to Taylor [9,10], the Nusselt number can be expressed as

$$Nu = 0.023Re^{0.8}Pr^{0.65}(T_w/T_h)^{-c}$$
 (1)

where c = 0.57 - [1.59/(z/D)]. Based on this correlation, in which

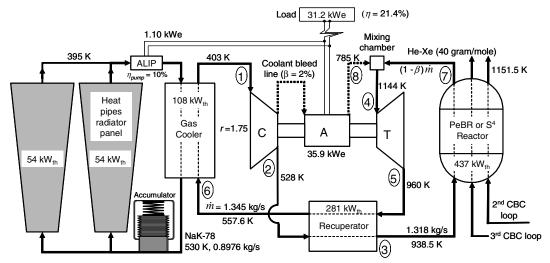


Fig. 2 Present space nuclear reactor power system with three CBC loops and sectored nuclear reactor.

the properties are evaluated at the gas bulk temperature, the heat transfer coefficient can be expressed as

$$h = 0.023 \underbrace{(T_w/T_b)^{-c} \dot{N}^{0.8}}_{\text{operation}} \cdot \underbrace{[D^{-0.2}A^{-0.8}]}_{\text{geometry}} \cdot \underbrace{[(M^{0.8}k^{0.35}C_p^{0.65})/\mu^{0.15}]}_{\text{gas properties}}$$
(2)

The first term on the right-hand side of Eq. (2) represents the dependence of the heat transfer coefficient on the operating temperatures of the wall and the bulk of flowing gas and its molar flow rate. The second term represents the dependence of the heat transfer coefficient on the effective diameter and cross-sectional area of the flow channels, and the third term represents the physical properties of the flowing gas or binary gas mixture. Thus, for the same operating conditions and geometry, the heat transfer coefficient can be expressed solely in terms of the gas thermal and transport properties as

$$h \propto \left(M^{0.8} k^{0.35} C_p^{0.65}\right) / \mu^{0.15}$$
 (3)

B. Relative Pressure Losses

Similarly, the pressure losses for a convective gas flow through circular ducts are expressed by the Fanning equation [11], as

$$\Delta P = 0.5a[L/(D^{1+b}A^{2-b})](\mu^b M^{2-b}/\rho)\dot{N}^{2-b}$$
 (4)

The gas density, expressed in terms of its temperature, pressure, and compressibility factor, is given as

$$\rho = (MP/R_gTZ) \tag{5}$$

Substituting Eq. (5) into Eq. (4) and rearranging the results gives the relative pressure losses as

$$(\Delta P/P) = 0.5R_g \underbrace{a(T/P^2)\dot{N}^{2-b}}_{\text{operation}} \underbrace{[L/(D^{1+b}A^{2-b})]}_{\text{geometry}} \underbrace{(\mu^b M^{1-b}Z)}_{\text{gas properties}}$$
(6)

For a turbulent gas flow, the coefficients $b \sim 0.2$ and $a \sim 0.184$ [11], thus, the relative pressure losses, in terms of the gas properties, are

$$(\Delta P/P) \propto (\mu^{0.2} M^{0.8} Z) \tag{7a}$$

At an operating pressure of 2.0 MPa in space nuclear reactor power systems with CBC engines, Fig. 3 shows that the compressibility factor at a reactor exit temperature of 1200 K is almost unity and independent of the molecular weight of the gas. However, at a coldleg temperature of 400 K (Fig. 2), Z for the binary mixtures of He-Xe

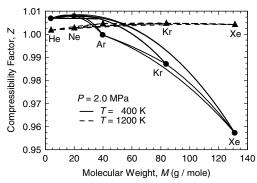


Fig. 3 Compressibility factors of noble gases and their binary mixtures.

and He-Kr decrease as the molecular weight of the mixture increases from 1.008 for pure helium (M=4 g/mol) to \sim 0.96 for pure Xe (M=131.3 g/mol) or \sim 0.987 for pure Kr (M=83.8 g/mol). Therefore, for all practical purposes, the compressibility factor for noble gases and their binary mixtures at a system pressure of 2.0 MPa is very close to unity and can be assumed constant; thus, Eq. (7a) can be simplified as

$$(\Delta P/P) \propto (\mu^{0.2} M^{0.8}) \tag{7b}$$

At a given temperature and mass flow rate, the pumping load in a component i in the CBC loop is directly proportional to the relative pressure losses in that component, $(\Delta P/P)_i$, and can be expressed as

$$\dot{W}_i = (\Delta P/P)_i (T_i Z_i) R_g \dot{N} \tag{8}$$

In this equation, T_i and Z_i are the gas bulk temperature and compressibility factor, and the relative pressure ratio $(\Delta P/P)_i$ is given by Eq. (6).

C. Aerodynamic Loading

The enthalpy rise in a centrifugal-flow compressor of a CBC engine is a useful indicator of the aerodynamic loading of the impeller blades. For given operating pressure and temperatures, the size and mass of the centrifugal-flow turbomachines used in space nuclear power systems would be expected to increase with increasing the tip radius of the compressor impeller blades, R_C . The aerodynamic loading of the blades is expressed as [12]

$$\lambda_C = \left(\Delta H_C / R_C^2 \omega^2\right) \tag{9}$$

It affects the thickness and mass of the blades and the hub. For the

same shaft rotational speed ω and aerodynamic loading of the impeller blades, the outer radius of the compressor and the size and mass of the turbomachine increase as ΔH_C increases. Thus, the aerodynamic loading, expressed in terms of ΔH_C [12], affects the size and mass of the centrifugal-flow Brayton engines, for which the compressor loading factor λ_C is typically less than 0.75, and for the turbine, $\lambda_T < 0.90$ (Table 1).

For a given ΔH_C , increasing the rotational speed ω and/or the tip radius of the compressor impeller blades decreases the aerodynamic loading. A large impeller tip radius requires thicker and heavier disk and blades, thus increasing the mass of the turbomachines. Conversely, for the same rotational speed, the tip radius of the compressor impeller blades and the size and mass of the CBC engine units decrease as ΔH_C decreases. The enthalpy rise in the compressor, expressed in terms of the compressor inlet temperature, polytropic efficiency, compression ratio, and the gas properties, is given as

$$\Delta H_C = (\hat{C}_P/M) T_{1,o} [r^{\frac{(\gamma - 1)}{\gamma \eta_C}} - 1] \propto (\hat{C}_P/M)$$
 (10)

Because the molar heat capacities of noble gases and binary mixtures at the inlet temperature to the compressor (400–450 K) are almost constant, for the same inlet temperature $T_{1,o}$, for the same compressor polytropic efficiency and compression ratio, the aerodynamic

loading is inversely proportional to the molecular weight of the gas working fluid [Eqs. (9) and (10)].

III. Transport Properties

Equations (3) and (7b) indicate that the most important transport properties of noble gases and binary mixtures are the thermal conductivity and the dynamic viscosity. The calculated [7] values at 2.0 MPa and typical cold-leg (400 K) and hot-leg (1200 K) temperatures in a CBC loop of a space reactor power system are compared in Figs. 4a and 4b.

A. Thermal Conductivity

Figure 4a shows that the thermal conductivities of the noble gases and binary mixtures nearly double as the temperature increases from 400 to 1200 K, but decrease significantly as the molecular weight increases. The thermal conductivities of the binary mixtures of the lowest-molecular-weight noble gases of He or Ne with heavier gases are higher than those of the pure gases of the same molecular weights. The binary mixture of He-Xe has the highest thermal conductivity of all the binary mixtures, which decreases from that of pure He to that of Xe as the molecular weight of the mixture increases from 4.003 to 131.29 g/mol. The thermal conductivities of the He-Xe mixtures of the same molecular weights of Ne (M=20 g/mol) and Ar

Table 1 Comparison of design and performance parameters of space reactor power systems with CBC energy conversion

Parameter	BRU [1]	SR-100G [13]	NASA-GRC [14]	Present system (Fig. 2)
System				
Working fluid, g/mol	He-Xe (83.8)	He-Xe (40)	He-Xe (40)	He-Xe (40)
CBC loops/engines operating (installed)	1/1(1)	2/1 (2)	2/2 (2)	3/3 (40)
Reactor hydraulics		Nonsectored	Nonsectored	Sectored (3)
Bleed-line fraction β	0.02	0.01	0.02	0.02
Recuperator effectiveness ε	0.95	0.767	0.95	0.95
Pressure-loss ratio $\Delta P/P$				
Recuperator hot leg	3.3%		1.5%	1.5%
Reactor (or heat exchanger, HX)	2.7%		2.7%	2.7%
Recuperator cold leg	1.7%		0.6%	0.6%
Gas cooler (or radiator)	0.5%		1.0%	1.0%
Effective pressure-loss coefficient π	0.920	0.96	0.943	0.943
Net system efficiency η , %	29.85	19.7	21.65	21.4
Net electrical power output, kWe	6.00	$1 \times 100 = 100$	$2 \times 50 = 100$	$3 \times 31.2 = 93.6$
Nuclear reactor (or heat exchanger)				
Exit temperature $T_{7,o}$, K (Fig. 2)	1144	1400	1150	1151.5
Inlet temperature $T_{3,o}$, K (Fig. 2)	918.3	1028	905	938.5
Thermal power Q_{RX} , kW _{th}	20.1	501.0	462.0	437.5
Compressor				
Polytropic efficiency η_C , %	80.0	84.7	82.4	83.0
Pressure ratio r	1.90	1.92	2.0	1.75
Inlet temperature $T_{1,o}$, K (Fig. 2)	294.4	514	400	403
Exit temperature $T_{2,o}$, K (Fig. 2)	409.4	695	555	528.0
Impeller diameter, cm	10.8	15.6		12.82
Enthalpy rise ΔH_C , kJ/kg	27.7	95.9	83.2	64.8
Aerodynamic loading λ_C	0.667	0.71		0.71
Turbine				
Polytropic efficiency η_T , %	87.0	90.0	87.3	87.5
Gas flow rate in turbine, kg/s	0.3611	1.30	1.85	1.345
Inlet temperature $T_{4,o}$, K (Fig. 2)	1144	1400	1150	1144
Exit temperature $T_{5,a}$, K (Fig. 2)	945.0	1127	924	960.0
Impeller diameter, cm	12.62	17.1		13.94
Enthalpy decrease ΔH_T , kJ/kg	50.1	143.8	118.8	95.6
Aerodynamic loading λ_T	0.886	0.886		0.886
Generator	0.000	0.000		0.000
Shaft rotational speed, rpm	36,000	45,000	45,000	45,000
Gross shaft power, kW	7.917	1 × 124.8	2×64.8	3×41.4
Thermodynamic cycle efficiency, %	39.4	24.57	28.06	28.4
Shaft mechanical efficiency η_M , %	87.7	84.4	81.22	86.7
Alternator (electrical) efficiency η_M , %	86.4	95.0	95.0	90.0
Heat-rejection radiator	00.1	75.0	75.0	70.0
Gas cooler inlet temperature $T_{6,\rho}$, K (Fig. 2)	436.7	801	581	557.6
Effective radiator surface area, m ²		61.0	186.4	203
Radiator surface emissivity		0.89	——	0.90
Effective space sink temperature, K		200	200	200
Total power rejected Q_{rad} , kW _{th}		400	365	324
roun power rejected grad, K to th		700	303	327

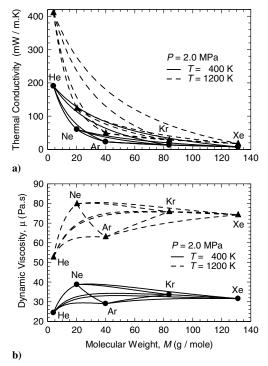


Fig. 4 Thermal conductivities and dynamic viscosities of noble gases and their binary mixtures.

(M = 40 g/mol) are two and four times higher than those of these pure gases.

B. Dynamic Viscosity

Figure 4b shows that the dynamic viscosities of the noble gases and binary mixtures almost double as the temperature increases from 400 to 1200 K, regardless of the value of the molecular weight. The dynamic viscosities of the binary mixtures of He with Ne or Ar increase from the lowest value of pure He to those of Ne and Ar as the

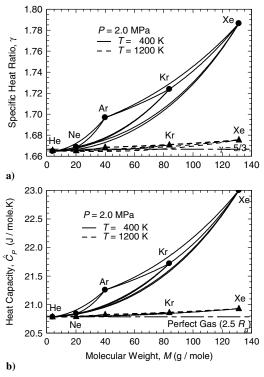


Fig. 5 Specific heat ratios and molar heat capacities of noble gases and their binary mixtures.

molecular weight of the mixture increases. Pure Ne has the highest dynamic viscosity, followed by Kr, Xe, Ar, and He. Thus, when mixing He with Ne or with any of the higher-molecular-weight pure noble gases of Ar, Kr, or Xe, the dynamic viscosity of the resulting binary mixture decreases as the molecular weight decreases to that of He or increases to that of the heavier noble gas in the mixture as the molecular weight of the binary mixture increases (Fig. 4b).

C. Heat Capacity

For the operating pressure of interest in space reactor power systems (2.0 MPa), the specific heat ratios of the noble gases and binary mixtures at 1200 K are essentially constant and equal (to within less than 1%) to that of a perfect gas, $\gamma = 5/3$ (Fig. 5a). At 400 K, the specific heat ratio of the binary mixture of He-Xe is the lowest of all other mixtures and increases from that of He (1.667) to that of Xe (1.784) as the molecular weight of the mixture increases. Similarly, at this temperature, γ for the He-Kr binary mixture increases from that of He to that of Kr (1.725), with increasing molecular weight of the mixture. Figure 5b shows that the molar heat capacities of the noble gases and binary mixtures, $\hat{C}_P = R_g \times \gamma /$ $(\gamma - 1)$, change in a similar fashion as γ , with increasing molecular weight. At 1200 K, it is almost constant (\sim 20.8 J/mol·K) and independent of the molecular weight. However, at 400 K, the molar heat capacity of the He-Xe binary mixture is the lowest of all binary mixtures and increases as the molecular weight of the mixture increases, from $\sim 20.8 \text{ J/mol} \cdot \text{K}$ for He to 23.0 J/mol $\cdot \text{K}$ for Xe.

D. Prandtl Number

The Prandtl number is the ratio of the thermal diffusivity to the kinematic viscosity of the gas working fluid (α/ν) , thus its value strongly affects the heat transfer coefficient [Eq. (1)]. Figure 6 compares the Prandtl numbers for the pure noble gases and binary mixtures. For pure gases, the Prandtl number is almost independent of temperature and molecular weight, increasing from ~ 0.67 for He to 0.7 for Xe. Conversely, the Prandtl numbers of the binary mixtures decrease rapidly with increasing molecular weight, below those of the lightest gases in the mixtures, to minimums, then rise to the Prandtl numbers of the heaviest gases in the mixtures. The minimum Prandtl numbers decrease as the molecular weight of the heavy gas in the binary mixtures increases and/or the molecular weight of the light gas in the mixtures decreases (Fig. 6). Thus, the Prandtl numbers of the binary mixtures are always lower than those of the pure noble gases in the mixtures. The binary mixture of He-Xe has the lowest Prandtl number, with a minimum of ~ 0.218 at a molecular weight of \sim 50 g/mol (36 mol % Xe and 64 mol % He).

As mentioned earlier, the transport and thermodynamic properties of the noble gases and binary mixtures are accurately calculated using semi-empirical relations, developed using an extended kinetic theory approach and the two-parameter law of corresponding states. The obtained relations are successfully benchmarked against an extensive experimental database [7]. For the temperatures and pressure of interest in space nuclear power systems with a CBC (400–1200 K and pressure less than or equal to 2.0 MPa), the errors in the dynamic viscosities of pure gases and binary mixtures are less

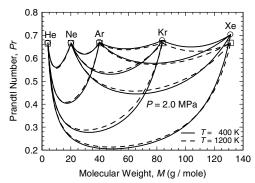


Fig. 6 Prandtl number of noble gases and their binary mixtures.

than 2.5 and 3.7%, respectively, compared to experimental data. The calculated thermal conductivities are within 5% of the great majority (93%) of the experimental values in the database, and the calculated densities and compressibility factors for the pure noble gases are within 0.2% of the values recommended by the National Institute of Standards and Technology (NIST) [7]. Similarly, the specific heat capacities and ratios are within 0.2% of the values recommended by NIST [7].

IV. Relative Performance

The calculated transport properties of the pure noble gases and binary mixtures (Figs. 3–6) are used to calculate the relative values of their heat transfer coefficients and pressure losses, and the aerodynamic loading of the compressor impeller blades at typical operating conditions in space nuclear reactor power systems with CBC engines (i.e., 2.0 MPa and temperatures of 400 and 1200 K). The following subsections present and discuss the results.

A. Normalized Pressure Losses

The pumping load of the compressor equals the sum of the pressure losses in the piping segments and the various components in the CBC loop (for example, see Fig. 2). These are the recuperator, the nuclear reactor, and the gas cooler that couples the CBC gas loop to the NaK-78 water-heat-pipe radiator panels [8]. In Fig. 7, the calculated pressure losses [Eq. (7a)] are divided by those of pure helium for the same geometrical dimensions and operation conditions, molar flow rate, temperature, and pressure [Eq. (6)]. For the pure noble gases and binary mixtures with molecular weights less than 80 g/mol, the normalized pressure losses are almost independent of the gas composition and temperature, but increase rapidly with increasing molecular weight. As delineated in Fig. 7, for Ar or the binary gas mixtures of the same molecular weight of 40 g/mol, the normalized pressure losses are more than 6.5 times those with pure helium. The latter is the coolant of choice for gas-cooled nuclear reactors in terrestrial electrical power plants with CBC engines [5,6].

It is worth noting that the differences between the relative pressure losses with pure noble gases and their binary mixtures of the same molecular weights are insignificant. For pure Xe and its binary mixtures with molecular weights greater than 80, the relative pressure losses increase by up to 13% as the temperature increases from 400 to 1200 K. The relative pressure losses are the highest with pure Xe. At 400 and 1200 K, these pressure losses are 15.6 and 17.6 times those with pure helium. However, for practical application in space nuclear reactor power systems with CBC engines, for which the molecular weight of the noble-gas binary mixtures, as discussed later, is lower than that of Kr, it is safe to assume that the normalized pressure losses and the pumping load of the compressor are independent of the gas temperature. Thus, the pressure losses in a CBC loop (Fig. 2) are accurate when calculated at the average gas temperature in the loop or the nuclear reactor. Based solely on lowpressure-losses consideration in a CBC loop, helium is by far the best and Xe is by far the worst choice of a working fluid.

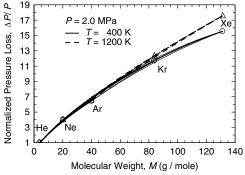


Fig. 7 Normalized pressure losses for noble gases and their binary mixtures.

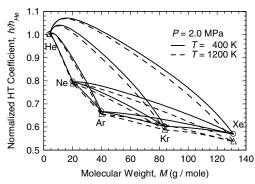


Fig. 8 Normalized heat transfer coefficients for noble gases and their binary mixtures.

B. Normalized Heat Transfer Coefficient

The heat transfer coefficient [Eq. (3)], normalized to that of pure helium, is indicative of the heat transfer surface area and the size and mass of the nuclear reactor, the radiator gas cooler (Fig. 2), and the recuperator [8]. Therefore, to reduce the size and mass of these heat transfer components, one would select a working fluid with the highest heat transfer coefficient. This favors lower-molecular-weight pure noble gas, particularly helium, and some binary gas mixtures in a narrow range of molecular weights (Fig. 8).

Figure 8 plots the normalized heat transfer coefficients for the noble gases and binary mixtures at typical operating conditions in a space reactor power system with a CBC. Helium has the highest heat transfer coefficient of all pure noble gases. The heat transfer coefficient for the pure noble gases decreases as the molecular weight increases. Thus, the lowest heat transfer coefficient is that of pure Xe. When mixing He with the higher-molecular-weight gases of Ar, Kr, and Xe, however, the resulting binary mixtures have higher heat transfer coefficients than pure He when the molecular weight of the mixture is less than or equal to 9, 25, and 45 g/mol, respectively (Fig. 8). Increasing the molecular weight of these mixtures beyond these values causes their heat transfer coefficients to decrease below that of pure He, but remain higher than the heat transfer coefficients of the higher-molecular-weight pure gas in the mixture.

The binary mixture of He-Xe has a significantly higher heat transfer coefficient than any of the other binary mixtures. Thus, for space nuclear power system applications, a binary mixture of He-Xe with a molecular weight of less than or equal to 40–45 g/mol has the highest heat transfer coefficient and thus, is the best choice for minimizing the heat transfer areas and size of the reactor, recuperator, and radiator gas cooler [8] (Fig. 2). The binary mixtures of He-Xe with molecular weights of 40–45 g/mol have almost the same heat transfer coefficients as pure helium, but as shown later, the resulting aerodynamic loading of the compressor impeller blades in the turbomachines is significantly lower than with helium. The maximum heat transfer coefficient of the He-Xe mixture occurs at a molecular weight of 15 g/mol; it is only 7% higher than those of pure helium and the binary mixture with a molecular weight of 40 g/mol (Fig. 8).

C. Normalized Aerodynamic Loading

Because the molar heat capacity is nearly constant, λ_C [Eqs. (9) and (10)] is essentially inversely proportional to the gas molecular weight and nearly independent of the mixture composition. Figure 9 plots the normalized heat transfer coefficient and the aerodynamic loading of the compressor impeller blades for the binary mixtures of He-Xe and He-Kr vs the molecular weight of these mixtures. Increasing the molecular weight of the pure gases or binary mixtures significantly decreases the aerodynamic loading of the blades and the size and mass of the turbomachines. The latter is possible with practically no compromise in the heat transfer coefficient compared with that of pure helium, if the molecular weight of the He-Xe binary mixture \sim 40 g/mol. At higher molecular weights, however, the reduction in the aerodynamic loading comes at the expense of decreasing the heat transfer coefficient, which in turn increases the

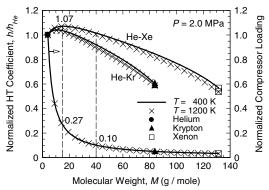


Fig. 9 Normalized compressor loadings and heat transfer coefficients for He-Xe and He-Kr gas mixtures.

heat transfer areas and the size and mass of the reactor, gas cooler, and recuperator in the CBC loop (Fig. 2). For the binary mixtures of He-Kr with molecular weights of 10–20 g/mol and He-Xe with molecular weights of 15–40 g/mol, depending on the temperature, there is a significant decrease in the aerodynamic loading, whereas the heat transfer coefficients are equal to or slightly higher than that of pure helium. However, because the heat transfer coefficient of the binary mixture of He-Xe is higher than that of the He-Kr mixture, it is a better choice for a working fluid in space nuclear reactor power systems with a CBC.

For the binary mixture of He-Kr with a molecular weight of 20 g/ mol, the heat transfer coefficient is almost the same as that of pure helium, but the normalized aerodynamic loading of the compressor impeller blades is 20% of that with pure helium. With this He-Kr binary mixture, the relative pressure losses in the loop are four time those with pure He. The He-Xe mixture with a molecular weight of 40 g/mol (or 28 mol % Xe and 72 mol % He) is a very attractive working fluid, because the blade's aerodynamic loading is only 10% of that with pure helium and the heat transfer coefficient is the same as He. However, the relative pressure losses with this working fluid are 6.5 times those with pure He. Decreasing the molecular weight of the He-Xe mixture to 15 g/mol (8.6 mol % Xe and 91.4 mol % He) increases the heat transfer coefficient by \sim 7% and decreases the relative pressure losses from 6.5 to 3 times those with helium, but at the same time nearly triples the aerodynamic loading of the compressor blades to 27% of that with pure helium. Note that the temperature only slightly affects the normalized heat transfer coefficients of the pure noble gases and binary mixtures (Figs. 8 and 9).

V. Space Nuclear Power System with Three CBC Loops

This section examines the impact of using He-Xe binary mixtures with molecular weights of 15 and 40 g/mol as working fluids on the net peak efficiency and load electrical power of a space nuclear reactor power system. The molecular weight of 15 g/mol for the He-Xe binary-mixture working fluid corresponds to a heat transfer coefficient that is ~7% higher than that of pure helium (Figs. 8 and 9). A He-Xe gas mixture with a molecular weight of 40 g/mol has almost the same heat transfer coefficient as pure helium (Figs. 8 and 9). This section presents the performance parameters of the present 93.7-kWe space nuclear power system with three independent CBC loops and sectored gas-cooled reactor, which uses a He-Xe binary-mixture working fluid with a molecular weight of 40 g/mol (Fig. 2). The calculated net peak efficiency and load electrical power of this system are compared with those reported for other systems with similar and slightly different operation conditions [1,13,14].

A. System Description

Figure 2 presents the layout with the nominal full-power performance parameters of the present space nuclear power system. The reactor, cooled with a binary mixture of He-Xe (40 g/mol), is

divided into three identical sectors loaded with highly enriched nuclear fuel. The sectors are neutronically and thermally coupled, but hydraulically decoupled to avoid a single-point failure in reactor cooling. Each reactor sector provides thermal power to a separate CBC loop, with a recuperator and a gas cooler, served by two separate radiator panels. The power system continues to operate with only two CBC engines working, but at a lower reactor power, following a failure of the turbomachine, a loss of cooling, or a break in one of the loops. The fission power generated in the reactor sector connected to the failed CBC loop is transported by conduction and/or radiation to the dividers with the two adjacent sectors and is removed by forced convection of the circulating gas in these sectors. Examples of sectored gas-cooled space reactors are the pellet bed reactor (PeBR in Fig. 2) and the submersion-subcritical safe space S⁴ reactor [3,15–18].

The effectiveness of the gas cooler and the recuperator in each CBC loop (Fig. 2) of the present system is 0.97 and 0.95. The flow of the molten sodium-potassium alloy, NaK-78 (78 wt % Na and 22 wt % K) in the secondary loops transports the thermal power extracted from the He-Xe in the gas coolers to the panels of the water-heat-pipe radiator. The panels, two per CBC loop, are hydraulically coupled in parallel to reduce pressure losses and the inventory of the NaK-78 liquid in the secondary loops [8]. The NaK coolant enters the radiator segments at 530 K and exits at 395 K. Each of the three NaK loops in the space power system (Fig. 2) has a bellows-type accumulator to accommodate the changes in the volume of liquid NaK during transient operation and system startup [19]. More details on the radiator panels' design and operation can be found elsewhere [8].

Alternative linear induction pumps (ALIPs) circulate the liquid NaK-78 in the secondary loops, one for each loop. Each CBC engine delivers a net electric power to the load of 31.2 kWe, after accounting for 10% transmission and inversion losses and the electrical power supplied to the ALIP. Each radiator panel nominally rejects 54 kW_{th}, and the total thermal power generated in the three sectors of the reactor is 437 kW_{th}. The binary-mixture working fluid of He-Xe (40 g/mol) enters the reactor sectors at 938.5 K and exits at 1151.5 K. The net peak conversion efficiency of the power system shown in Fig. 2 is 21.4%. The operating parameters in Fig. 2 account for the mechanical losses in the bearings, the electrical losses in the alternators windings, and the electromagnetic losses in the electrical generators of the turbomachines (Table 1).

Figure 10 shows the present space power system fully deployed. It has six water-heat-pipe radiator panels, two for each CBC loop, and a radiation shadow shield placed behind the reactor to protect the payload and instrumentation and control equipment from the reactor's fast neutrons and gamma radiation. Each radiator panel consists of a forward fixed segment and two rear deployable segments, and in the stowed configuration, the rear segments fold over the six forward fixed segments [8]. The stowed power system has a maximum height of 8.0 m and fits within the fairing envelop of the DELTA-IV heavy launch vehicle, configured in a dual-manifest arrangement [20]. The effective view factors of the radiator, accounting for the radiation from the outer and the inner surfaces, are 1.06 for the forward fixed segments and 1.273 for the rear segments. The view factors of the inner surfaces assume 20% obstruction by the payload boom and the components within the radiator cavity. With a 30-deg cone angle and a minor diameter of 1.08 m, the developed radiator has an external surface area of 168.9 m² and an effective heat-rejection area of 203 m².

The operating temperatures shown in Fig. 2 are similar to those of the NASA John H. Glenn Research Center (NASA–GRC) reference power system for a net electric power to the load of 100 kWe [14]. The technology of centrifugal-flow, single-shaft CBC engines for turbine inlet and exit temperatures of 1144 and 960 K is quite mature, with extensive testing for more than 30,000 h; only engineering implementation is required [3]. The He-Xe working fluid (40 g/mol) enters the compressor at 403 K and exits at 528 K. Two percent of the He-Xe gas flow is bled off downstream of the compressor to cool the bearings and the alternator rotor (Fig. 2); it mixes with that exiting the reactor at 1151.5 K. The resulting inlet temperature to the turbine is 1144 K.

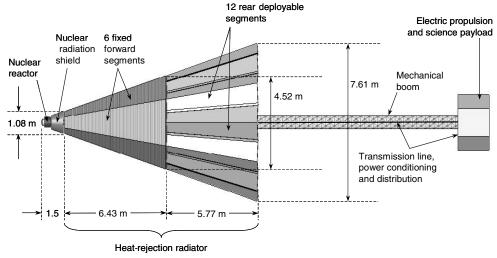


Fig. 10 Present space reactor power system with the radiator fully deployed.

B. System Performance

This section summaries the performance parameters of the present space reactor power system (Figs. 2 and 10), calculated using the model described in the Appendix. The system performance parameters are calculated for two He-Xe binary mixtures with molecular weights of 15 and 40 g/mol and for pure helium. The analysis assumes the same dimensions for the piping, the recuperator, the gas cooler, the reactor, and the radiator; the same radiator panel heat-rejection load of 54 W_{th} (a total radiator heat-rejection capability of 324 kW_{th}); and the same compressor and turbine inlet temperatures of 403 and 1144 K, but varied the pressure ratio of the compressor. The analysis also assumes a compressor polytropic efficiency $\eta_C = 0.83$, turbine polytropic efficiency $\eta_T = 0.875$, mechanical efficiency of the turbomachines $\eta_M = 0.867$, and an electrical generator efficiency $\eta_G = 0.90$, and

it accounts for the electrical power supplied to the ALIPs in the NaK-78 secondary loops (3.3 kWe).

Figures 11a and 11b show that the net peak efficiency and peakload electrical power of the present system (Figs. 2 and 10) occur at the same compressor pressure ratio. Increasing the molecular weight of the working fluid lowers the peak efficiency and the peak electrical power and shifts them to a higher compressor pressure ratio. This is because of the large increase in the pressure losses with increasing molecular weight of the gas working fluid (Fig. 7). The space power system in Figs. 2 and 10 is initially designed for a He-Xe gas working fluid with a molecular weight of 40 g/mol and total relative pressure losses in each CBC loop of 5.8% (see Table 1), corresponding to a pressure-loss factor in the CBC loops of $\pi=0.943$. When a He-Xe mixture with a lower molecular weight (15 g/mol) is used instead, the effectiveness of the recuperator increases from 0.95 to 0.9532 and π

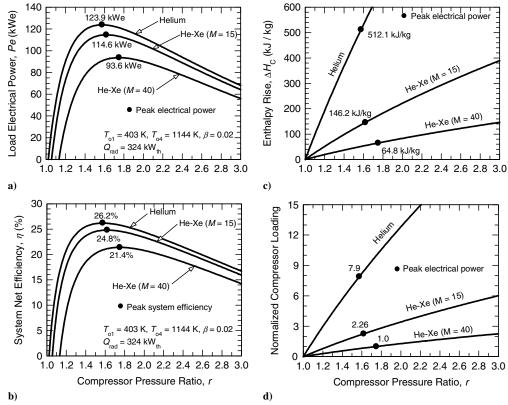


Fig. 11 Effect of compressor pressure ratio on the present space power system performance.

increases to 0.9747. These changes are caused by the increase in the heat transfer coefficient (\sim 7%) and the large decrease in the pressure losses in the loop (Figs. 7 and 8). When pure He is used as the working fluid in the CBC loops of the present system (Figs. 2 and 10), the recuperator effectiveness ($\varepsilon = 0.95$) is the same as that with He-Xe (40 g/mol), but the pressure-loss factor in the CBC loops increases to 0.99. This is because the heat transfer coefficient of He is identical to that of He-Xe (40 g/mol), but the pressure losses are 6.5 times lower for He. As a result, using a helium working fluid increases both the net system peak efficiency and load electrical power to 26.2% and 123.9 kWe, which are the highest. These correspond to a net-load electrical power of 41.3 kWe for each of the three CBC loops and a total reactor thermal power of 473 kW_{th}, a compressor pressure ratio of 1.58, and an enthalpy rise in the compressor of $\Delta H_C = 512.1 \text{ kJ/kg}$. On the other hand, the aerodynamic loading of the compressor impeller with He working fluid is 7.9 times that with the binary mixture of He-Xe (40 g/mol). It is also 3.5 times the aerodynamic loading of the compressor impeller with a He-Xe binary mixture having a molecular weight of 15 g/mol (Figs. 11c and 11d).

To decrease the aerodynamic loading of the compressor impeller with helium working fluid, it is necessary to increase R_C and/or ω ; the same is true for the turbine blades of the CBC engines (Fig. 1). Increasing the radii of the compressor impeller and turbine blades increases the size and mass of the turbomachines, whereas increasing the shaft rotational speed increases the impeller's centrifugal stresses and the frequency of the load electrical power. Therefore, lowermass turbomachines favor using a He-Xe binary mixture with a molecular weight of 40 g/mol as the working fluid in space reactor power systems with CBC loops (Figs. 2 and 10). On the other hand, with such working fluid, the system's net peak efficiency decreases from 26.2% with helium to 21.4%, and the optimum compressor pressure ratio increases from 1.58 with He to 1.76. Consequently, the net peak-load electrical power decreases by 24.5% to 93.6 kWe, instead of 123.9 kWe with He working fluid, but the reactor thermal power also decreases by 8% from 473 kW_{th} with He working fluid to 437 kW_{th}. In addition to slightly decreasing the masses of the reactor and the radiation shadow shield (by less than 6%), using He-Xe (40 g/ mol) working fluid instead of He decreases the rate of fuel burnup and reactivity depletion in the nuclear reactor, potentially increasing the operation lifetime of the power system by \sim 8% for use on longerduration missions.

The low system efficiency of 21.4% with He-Xe working fluid (40 g/mol) compared with 26.2% with He is essentially caused by the increased pressure losses in the CBC loops, 6.5 times those with He. This decrease in system efficiency could be alleviated by increasing the diameter of the piping and the flow channels in the heat transfer components, which reduces the pressure losses in the CBC loops at the expense of increasing the volume and mass of these components. The optimum loop pressure-loss factor would be typically selected to minimize the specific mass (kg/kWe) of the integrated space nuclear power system. Such system-optimization analysis, however, is beyond the scope of the present work.

C. Performance Comparison with Other Systems

The calculated performance parameters of the present space reactor power system (Figs. 2 and 10) are compared in Table 1 with those reported by other investigators for similar systems with direct CBC loops and heat-pipe radiators, but nonsectored nuclear reactors [1,13,14]. These systems have either one CBC loop or two loops, each with a CBC engine. In the SR-100G power system, one of the two CBC engines operates at full power, whereas the other is kept on standby in case the first engine fails [13]. In the NASA–GRC system, the two CBC engines operate nominally at 50% of their design power rating, and if one fails, the remaining CBC engine operates at its nominal design power [14]. Although these options of coupling multiple CBC engines to a nuclear reactor provide some redundancy in energy conversion, they complicate the piping and the integration of the power system. In addition, they could nearly double the mass of the energy-conversion system and, most importantly, they do not

eliminate single-point failures in the reactor cooling and energy conversion, contrary to the present space power system (Figs. 2 and 10).

The Brayton rotating unit (BRU), a single-shaft, radial-flow CBC engine [1] (Fig. 1) developed by NASA, was manufactured and tested successfully for extended periods (greater than 30,000 h). Operating on a gas bearing, the BRU generates 2.25-10.5 kWe, 3phase, 1200-Hz electrical power using a He-Xe binary-mixture working fluid with a molecular weight of 83.8 g/mol (same as pure Kr). The BRU was designed to operate at a rotational speed of 36,000 rpm and turbine inlet temperature of 1144 K. The 10.5-kWe unit in Fig. 1, complete with insulation and interface ducting, weighs \sim 80 kg (specific power \sim 7.6 kg/kWe). Table 1 compares the operation and performance parameters of the BRU unit with those calculated for the space nuclear power systems developed by Rockwell International (SR-100G [13]) and by NASA-GRC [14], along with those of the present system (Figs. 2 and 10). The performance parameters of these systems are based solely on preliminary design and operation analysis, not actual hardware development and testing of the CBC units. Note that the SR-100G, NASA-GRC, and the present power systems use the He-Xe binarymixture working fluid with a molecular weight of 40 g/mol, lower than that of the BRU (83.8 g/mol) (Table 1). The heavier binary mixture decreases the enthalpy changes in the compressor and the turbine units and/or their size and mass, but at the expense of operating at a 15% lower heat transfer coefficient than the He-Xe binary mixture with a molecular weight of 40 g/mol (Fig. 8).

The aerodynamic loading of the turbine blades in the BRU and the CBC engines of both the SR-100G and the present system is the same (0.886). The loading of the compressor impeller in the BRU of 0.667 is slightly smaller than in the CBC units of the SR-100G and the present power systems (0.71). The aerodynamic loading of the turbomachines in the NASA-GRC engines could not be found in the open literature. The shaft rotational speed in all three power systems in Table 1 is the same (45,000 rpm) and higher than that of the BRU (36,000 rpm). The SR-100G power system operates at significantly higher reactor and radiator temperatures than the other two systems. It employs a potassium-heat-pipe radiator with an effective surface area of only 61 m² and generates a nominal-load electrical power of 100 kWe. Both the present and NASA-GRC power systems employ water-heat-pipe radiators having effective areas of 203 and 186.4 m² and generate net-load electrical powers of 93.6 and 100 kWe, respectively. The specific area for the radiator in the SR-100G power system is 0.153 m²/kW_{th}, compared with 0.511 m²/kW_{th} in the NASA-GRC power system and 0.627 m²/kW_{th} in the present system. It is worth noting that this value for the present system is based on a detailed design and performance analysis of the NaK-78 water-heat-pipe radiator [8], whereas the specific areas of the radiators for the SR-100G and NASA-GRC systems are best estimates. The BRU net conversion efficiency (29.85%) is significantly higher than those of the three power systems in Table 1, because the compressor and the radiator inlet temperatures (294.4 and 436.7 K) are much lower. Such lower temperatures will significantly increase the specific area of the radiator in power systems with these BRU turbomachines.

VI. Conclusions

This paper examined the suitability of pure noble gases and binary mixtures as working fluids in space nuclear reactor power systems with a CBC and the effects of temperature and molecular weight on the thermodynamic and transport properties at a typical operating pressure of 2.0 MPa in space power systems with gas-cooled nuclear reactors and CBC engines. The calculated performance parameters of a space nuclear power system with no single-point failures in reactor cooling and CBC energy conversion, including the net peak efficiency and load electrical power, are compared with those reported by other investigators for similar systems.

The heat transfer coefficients of the binary mixtures of He-Xe and He-Kr increase slightly above that of pure helium as the molecular weights increase to certain values, then decrease rapidly with

increasing molecular weight. For the binary mixtures of He-Ar and He-Ne, the heat transfer coefficients decrease much faster than those of He-Xe and He-Kr with increasing molecular weight, thus these mixtures are excluded from further consideration. The binary mixtures of He-Xe have the highest heat transfer coefficient, followed by that of He-Kr. For a He-Kr binary mixture with a molecular weight of 20 g/mol, the heat transfer coefficient is almost the same as that of pure helium, but the aerodynamic loading of the compressor impeller and turbine blades is only 20% of that with pure helium. On the other hand, with a He-Xe binary mixture having a molecular weight of 40 g/mol (28 mol % Xe and 72 mol % He), the aerodynamic loading is only 10% of that with pure helium and the heat transfer coefficient is the same as He. Decreasing the molecular weight of the He-Xe binary mixture to 15 (8.6 mol % Xe and 91.4 mol % He) increases the relative heat transfer coefficient by \sim 7% compared with He, but almost triples the aerodynamic loading in the turbomachines.

Although He has the best transport properties of all noble gases and results in the highest system efficiency and load electric power, its low molecular weight significantly increases the aerodynamic loading and the size and mass of the single-shaft, centrifugal-flow turbomachines used in space reactor power systems. The binary mixtures of He with the heavier noble gases of Kr and Xe with molecular weights less than or equal to 22 and 40 g/mol, respectively, have slightly higher heat transfer coefficients than pure He, but the relative pressure losses in the CBC loops are up to 3 and 6.5 times those with He. These high pressure losses decrease the efficiency and load electrical power of space power systems, but could be reduced by increasing the equivalent diameter of the piping and the flow channels in heat-exchange units of the system (e.g., the reactor, recuperators, and gas coolers).

Given the strong emphasis in space nuclear reactor power systems on low mass and volume, the binary mixture of He-Xe with a molecular weight of 40 g/mol is the recommended working fluid for these systems with CBC energy conversion. It has the same heat transfer coefficient as He, whereas the induced aerodynamic loading of the compressor impeller and turbine blades is only 10% of that with He. Such low aerodynamic loading translates into smaller size and mass for the turbomachines and potentially lower system mass and stowed volume.

Appendix: Constituent Equations for Calculating Performance Parameters

The noble gases and binary mixtures are assumed to behave like a perfect gas, with a constant molar specific heat, a compressibility factor of unity, and a specific heat ratio of $\gamma=5/3$. These assumptions are justified based on the results in this paper of the thermodynamic and transport properties at the operating temperatures and pressure of 2.0 MPa in space power systems with CBC engines. These results are valid for noble gases and binary mixtures with a molecular weight of less than or equal to 40 g/mol (Figs. 3 and 5). The mechanical power developed in the turbine (\dot{W}_T) and that used in the compressor (\dot{W}_C) of a single-shaft, centrifugal-flow CBC engine can be expressed in terms of the molar flow rate, molar specific heat, and turbine and compressor operating temperatures (Fig. 2) as

$$\dot{W}_T = \dot{N}\hat{C}_P T_{1,o} \theta \left(1 - \frac{T_{5,o}}{T_4} \right) \tag{A1a}$$

$$\dot{W}_{C} = \dot{N}\hat{C}_{P}T_{1,o}\left(\frac{T_{2,o}}{T_{1,o}} - 1\right)$$
 (A1b)

The temperature ratios in these equations are expressed in terms of the cycle pressure ratio and the turbine and compressor polytropic efficiencies as [21]

$$\frac{T_{5,o}}{T_{4,o}} = \left(\frac{P_{4,o}}{P_{5,o}}\right)^{\left(\frac{1-\gamma}{\gamma}\right)\eta_T} = (r\pi)^{\left(\frac{1-\gamma}{\gamma}\right)\eta_T} \tag{A2a}$$

$$\frac{T_{2,o}}{T_{1,o}} = \left(\frac{P_{2,o}}{P_{1,o}}\right)^{\left(\frac{\gamma-1}{\gamma\eta_C}\right)} = r^{\left(\frac{\gamma-1}{\gamma\eta_C}\right)}$$
(A2b)

An expression for the pressure-loss factor is developed at the end of this Appendix. Thus, the net-load electrical power is given as

$$Pe = (\dot{W}_T - \dot{W}_C)\eta_M\eta_G - Pe_{ALIP} \tag{A3}$$

In the present space power system (Figs. 2 and 10), each ALIP consumes 1.1 kWe. As shown in Fig. 2, a fraction β of the working fluid is bled off at the exit of the compressor to cool the bearings and alternator. Assuming that the heat removed by bled-off gas equals the electrical losses in the alternator, the gas temperature to the mixing chamber, with the gas exiting the nuclear reactor, is given by

$$T_{8,o} = T_{2,o} + \frac{(\dot{W}_T - \dot{W}_C)\eta_M(1 - \eta_G)}{\beta \dot{N} \hat{C}_p}$$
 (A4)

Thus, for a given turbine inlet temperature $T_{4,o}$ (1144 K), the temperature of the working fluid exiting the nuclear reactor sectors (Fig. 2) is given as

$$T_{7,o} = \left(\frac{T_{4,o} - \beta T_{8,o}}{1 - \beta}\right) \tag{A5}$$

The reactor inlet temperature $T_{3,o}$ is expressed in terms of the recuperator effectiveness as

$$T_{3,o} = T_{2,o} + \varepsilon (T_{5,o} - T_{2,o})$$
 (A6)

The recuperator effectiveness can be written as a function of the exponential coefficient as

$$\varepsilon = \left(\frac{1 - \alpha}{1 - \alpha \cdot (1 - \beta)}\right) \tag{A7}$$

For a countercurrent flow in the recuperator, the coefficient α is expressed as [22]

$$\alpha = \left(\frac{T_{5,o} - T_{3,o}}{T_{6,o} - T_{2,o}}\right) = \exp\left[-\left(\frac{A_o U_o}{\dot{N} \hat{C}_p}\right) \left(\frac{\beta}{1 - \beta}\right)\right]$$
(A8)

The overall heat transfer coefficient for the recuperator is essentially proportional to the gas convective heat transfer coefficient, because the thermal resistance of the wall separating the two gas flows, compared with those of the flowing gas, is negligible. Consequently, for the same effective heat transfer area and flow conditions \dot{N} and β , α can be written in terms of that for a power system with helium working fluid as

$$\alpha = \exp\left[-\left(\frac{A_o U_o^{\text{He}} \cdot h/h_{\text{He}}}{\dot{N}\hat{C}_p}\right) \left(\frac{\beta}{1-\beta}\right)\right] = \exp[\ln(\alpha_{\text{He}}) (h/h_{\text{He}})]$$
(A9)

The term $h/h_{\rm He}$, the normalized heat transfer coefficient in Fig. 8, is a strong function of the molecular weight, but nearly independent of temperature and pressure. The coefficient for helium, $\alpha_{\rm He}$, is easily calculated as a function of the recuperator effectiveness by inverting Eq. (A7), to give

$$\alpha_{\text{He}} = \frac{(1 - \varepsilon_{\text{He}})}{1 - (1 - \beta)\varepsilon_{\text{He}}} \tag{A10}$$

The nuclear reactor thermal power is given by

$$Q_{RX} = (1 - \beta) \dot{N} \hat{C}_{n} (T_{7,n} - T_{3,n})$$
 (A11)

and the power system net efficiency is given as

$$\eta = (Pe/Q_{RX}) \tag{A12}$$

The molar flow rate is obtained from the energy balance in the gas cooler (Fig. 2), because the radiator heat rejection $Q_{\rm rad}$ is known from its detailed design:

$$\dot{N} = Q_{\text{rad}} / [\hat{C}_p (T_{6,o} - T_{1,o})]$$
 (A13)

The inlet temperature to the gas coolers, $T_{6,o}$, is obtained from the energy balance in the recuperator as (Fig. 2)

$$T_{6,o} = T_{5,o} - (1 - \beta)(T_{3,o} - T_{2,o}) \tag{A14}$$

The gas properties affect the values of π and ε for the CBC cycle. By definition, the pressure-loss factor can be written as (Fig. 2)

$$\pi = \left(\frac{P_{1,o}}{P_{2,o}}\right) \left(\frac{P_{4,o}}{P_{5,o}}\right) = \left(\frac{P_{3,o}}{P_{2,o}}\right) \left(\frac{P_{4,o}}{P_{3,o}}\right) \left(\frac{P_{6,o}}{P_{5,o}}\right) \left(\frac{P_{1,o}}{P_{6,o}}\right) \tag{A15}$$

The pressure ratio in a system component $\{i-j\}$ is related to the relative pressure losses $\Delta P/P$ given by Eq. (6) and can be expressed

$$\begin{split} \pi_{ij} &= \left(\frac{P_{j,o}}{P_{i,o}}\right) = \left(\frac{P_{i,o} - \Delta P_{ij}}{P_{i,o}}\right) = 1 - \left(\frac{\Delta P_{ij}}{P_{i,o}}\right)_{\text{He}} f \\ &= 1 - \left(1 - \pi_{ij}^{\text{He}}\right) f \end{split} \tag{A16}$$

In this equation, f is the normalized pressure loss delineated in Fig. 7 as a function of the molecular weight of the binary gas mixture. It is nearly independent of the gas composition, temperature, and pressure. Thus, the pressure-loss factor can be calculated as

$$\begin{split} \pi &= (\pi_{23} \quad \pi_{34} \quad \pi_{56} \quad \pi_{61}) = \left[1 - \left(1 - \pi_{23}^{\text{He}}\right) f\right] \\ &\times \left[1 - \left(1 - \pi_{34}^{\text{He}}\right) f\right] \left[1 - \left(1 - \pi_{56}^{\text{He}}\right) f\right] \left[1 - \left(1 - \pi_{61}^{\text{He}}\right) f\right] \end{split} \tag{A17}$$

Finally, making use of the definitions of ε and α , the energy balance in the recuperator gives the rate of heat transfer in it, Q_R , as

$$Q_R = (T_{5,o} - T_{6,o})\dot{N}\hat{C}_p = [(1 - \beta)(T_{3,o} - T_{2,o})]\dot{N}\hat{C}_p$$
 (A18)

Acknowledgment

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